

Population ageing and implications for labor market: case of Poland*

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Abstract

The objective of this study is to examine the relationship between population ageing and unemployment in Poland. Poland represents a particularly intriguing case for analysis within the context of the European Union, given its status as one of the member states with the lowest fertility rate and a notable trajectory of economic growth and significant labour market shifts over the past two decades. A New Keynesian OLG model with a realistic age structure, labour market frictions, sticky prices and aggregate shocks is employed to quantify the extent to which demographic changes over the past 20 years have contributed to the decline in the unemployment rate. While the reduction in unemployment in Poland over the past two decades has been largely attributed to shifts in the labour market, the ageing of the working population has been identified as a significant contributing factor, accounting for approximately 20% of the overall decline in unemployment. In addition, unemployment projections are presented for three age groups: young, prime-age and elderly up to 2040. Finally, the impact of population ageing on monetary policy has been examined, with a particular focus on the trade-off between unemployment deviation and GDP growth. The findings have significant policy implications, given the projected ageing of the Polish working population.

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1 Introduction

We examine the impact of population ageing on the level and volatility of unemployment in Poland. To this aim we employ a New Keynesian model with overlapping generations and labour market frictions with the evolving age structure of the working population. Poland is an interesting country to study for at least two reasons. First, Poland is one of the European Union member states with the lowest fertility rate and the most rapid population ageing process. This phenomenon has a significant impact on the relationship between young and elderly workers, as evidenced as presented in the Figure. 1. Since the mid-1990s, the proportion of the population of elderly workers (aged between 55 and 64) has exceeded that of young workers (aged 20 to 29). This discrepancy is still increasing, resulting in a gap between the two groups of 10 percentage points, which translates into over two million elderly workers outnumbering young workers in 2020. In 1993, the number of young people was equal to that of the elderly population, amounting to approximately 6.5 million for each group. In 2020, the elderly population was equal to 6.3 million, a similar level to that observed in 1993. However, at the same time, the number of young people decreased by over 2 million.

The reduction in the number of young workers, who are typically characterised by relatively higher unemployment than prime-age workers (reviewed by, e.g., Caliendo and Schmidl, 2016), and the increase in the number of elderly workers, who are typically characterised by low unemployment rates, leads us to hypothesise that unemployment may exhibit a secular decline in Poland. Makraski et al. (2024) demonstrate a significant correlation between population structure and unemployment rate. They estimate that the population change in the Eurozone between 1995 and 2015 contributed to a reduction in unemployment by approximately 1 percentage point. This article aims to assess the impact of population ageing on the Polish labour market.

Second, it is important to note that the Polish labour market has undergone significant changes since the 1990s. Poland is among the fastest-growing countries in the European Union. Figure 2 illustrates the growth of Poland's GDP, which has increased by approxi-

mately 160% since 1995. In comparison, the GDP growth of the United States has increased by approximately 80% and that of the Eurozone 45%. This economic development is reflected in the unemployment rate in Poland, which has reached a historically low level of approximately 3% since 2022. The unemployment rate in Poland has decreased significantly since 2000, from a level of 20% to a current rate that is lower than in the Eurozone since 2008 and lower than in the United States since 2018. This paper employs a longitudinal analysis of the Polish labour market, specifically focusing on the decline in unemployment across three distinct age groups: young (20-29), prime-age (30-54) and elderly (55-64) workers. Additionally, we estimate the changes in the labor market, as well as the impact of population structure shifts on the unemployment rate.

Contribution This article analyses the influence of demographic transition on the extent and variability of unemployment. In order to achieve this, we construct a New Keynesian model with overlapping generations and labour market frictions. Our model features are analogous to those of Makraski et al. (2024), and we also assume that workers vary by age and are not perfect substitutes in the production function. The model determines the unemployment and job creation rates through the search and matching function, following the approach set out by Mortensen and Pissarides (1994). The model combines aggregate shocks typical of the DSGE literature with realistic age structure characteristics for OLG models. This allows the model to perform two main exercises.

First, it is possible to examine the impact of demographic factors on unemployment. The composition of the workforce affects the unemployment rate through two distinct mechanisms. Firstly, the gradual decline in the proportion of young people in the total working-age population means that the group with the highest unemployment rate has lost its previously strong influence on the overall unemployment rate, which we call the composition effect. Secondly, the scarcity of workers may result in an increase in the value of young workers and, as a consequence, a reduction in the unemployment rate among young workers. In a general equilibrium model, firms will adjust to the demographic transition by adjusting the number of job openings, which will in turn trigger adjustments in the job search activity

of workers. This is the behavioural effect. Our model allows for the estimation of these two effects on the decrease in the unemployment rate in Poland. In addition, we present the projection of changes in unemployment up to 2040 for three working age groups: young, prime-age and older. The projection shows the changes in the unemployment rate in relation to the demographic changes. If the labour market were unchanged, the unemployment rate would fall by more than 1 percentage point by 2040 due to changes in population growth.

Second, we can set implications for monetary policy. We use a model of stochastic equilibria around a deterministic trend with age-specific search and matching frictions in the labour market, as proposed by Makarski et al. (2024). We aim to calibrate the model to the features of the Polish business cycle and to study the effects of changing population structure on both the level of unemployment (deterministic trend) and the volatility of unemployment (in a stochastic setup). In order to provide a meaningful quantification of the effects of demographics on unemployment levels and volatility, our model incorporates a number of key factors, including sticky prices and sticky wages, search and matching frictions, complementarity of workers in different age groups, investment adjustment costs and external habit formation.

We use the United Nations World Population Prospects 2019, covering years 1950-2020 for the evolution of population structure as Bielecki et al., 2020, to isolate the effects of demographics. The Labour Force Survey (BAEL) is employed to calibrate the labour market flows in Poland. The data illustrate a significant increase in the rate of job finding between the 2000s and 2015s. This increase was particularly noticeable for prime and elderly workers, while the rise for younger workers was more moderate. The individual-level data from BAEL demonstrate a Polish pattern comparable to that observed in the EU with regard to the separation rate. Initially, the rate was three times higher for the youth cohort than for the other age groups. This discrepancy narrowed considerably over the two-decade period, so that the separation rate for young people is now only two times higher than for other age groups. We have developed time-varying targets for age-specific labour market flows for two steady states: 2000s and 2015s.

Main results

Our results reveal several important findings. The reduction in unemployment over the past two decades can be attributed mainly to shifts in the labour market. The Polish labour market underwent significant transformations between 2000 and 2020, driven by Poland's accession to the European Union and the ongoing development of a free-market economy following the communist era. These shifts resulted in increased foreign investment, modernisation and greater worker mobility, which collectively reshaped Poland's economic landscape. Our analysis indicates that changes in the labour market account for more than 80% of the overall reduction in unemployment. The ageing process in Poland accounts for approximately 20% of this decline, with 1.9 percentage points resulting from behavioural effects and 0.7 percentage points due to compositional effects.

The most substantial decline in unemployment, amounting to approximately 20 percentage points, is observed among the younger workers. Of this, approximately 18 percentage points can be attributed to labour market changes, with the remaining 2 percentage points attributable to demographic shifts. Similarly, for the prime-age and older workers, demographic changes contribute to a reduction in unemployment by less than 2 percentage points.

Second, we analyze the impact of population aging on monetary policy, with a specific focus on the trade-off between deviations in unemployment and GDP growth. The results of our monetary frontier analysis indicate that in older populations, a reduction in the standard deviation of inflation can be achieved with a smaller increase in the standard deviation of unemployment. In other words, the model suggests that the sacrifice ratio is lower with older workforces. Consequently, monetary policy in ageing societies should prioritise inflation stabilisation over unemployment reduction, reflecting a more favourable trade-off between inflation and unemployment under a restrictive policy stance. Additionally, the impulse response function (IRF) analysis reveals that the economy returns to a steady state more quickly after an interest rate shock in aging populations, particularly regarding unemployment dynamics. These results suggest that optimal monetary policy in aging societies may adopt a more hawkish approach.

Our paper is structured as follows. The subsequent section presents the related literature. Section 3 introduces the model. Section 4 describes the calibration procedures. Section 5 describes the results for the level of unemployment, including both factual and counterfactual simulations. Section 6 presents the results for the volatility of unemployment and implications for macroeconomic policy. Section 7 concludes.

2 Related literature

Our paper is related to the three strands of literature. Firstly, we contribute to the literature regarding the differences in unemployment across various age group. The empirical literature concerning the youth unemployment rate reveals that it is necessary to differentiate between the various age groups entering the labour market. The youth unemployment rate exceeds the total unemployment rate (Bloom et al., 1988; O'higgins, 1997; Elder et al., 2006; Keese et al., 2013; Liotti, 2022) and is more cyclical (Freeman and Wise, 1982; Bell and Blanchflower, 2011; Dietrich and Möller, 2016). The literature posits that the difficulties associated with securing one's first stable employment are at the core of the problem (O'higgins, 2001; Elder et al., 2006). The substitution of workers of different ages in production is imperfect (Bloom et al., 1988). Our model reflects the differences across the age groups. With the demographic transition, there is a reduction in the number of young people entering the labour market, which will result in a corresponding reduction in their contribution to the total unemployment rate. Furthermore, the threshold for finding a job may be reduced if the demographic transition results in a shortage of workers. Due to imperfect substitution between workers of different ages, the reduction in the number of young people entering the market may impact the scale of unemployment for all the cohorts.

Second, empirical evidence on the role of demographic change on unemployment is scarce. Studies use the shift-share approach (Aaronson et al., 2015; Tuzemen, 2017), which implicitly assumes that the incidence of unemployment among age groups does not depend on their changing shares. Fallick and Foote (2022) account for age distribution and obtain sizable

effects of demographic change on unemployment. In our analysis, age structure and its changes match the data. Also, the drivers of unemployment (job finding rate and separations) reflect the evolution over time observed in the data. We use a model with realistic age structure to show how changes in the population affect unemployment. Our model is identical to Makarski et al. (2024), who use the OLG DSGE model with a realistic population structure to study how demographic change affects the Euro area labour market. They combine three types of models: search and matching, overlapping generations, and DSGE. The search and matching structure is based on Diamond (1982), Mortensen and Pissarides (1994), and Pissarides (2000). Hairault et al. (2007); Chéron et al. (2011, 2013), using a search and matching life-cycle model, characterize the optimal recruitment policies across age groups (young, prime age, and elderly workers) and find that the optimal profile for firing taxes and hiring subsidies would have to be hump-shaped in age.

Hairault et al. (2019), using the extension of the Chéron et al. (2013) model, analyze the cyclical properties of the labor market transition rates across age groups in the US. This literature holds the population structure constant (some of the existing papers introduce demographic shocks). The objective in these studies is to rationalize observed differences in labor market outcomes across the age groups.

Through the lens of OLG models, Corneo and Marquardt (2000), and Ono (2007; 2010; 2019) analyze the effect of social security on unemployment. De la Croix et al. (2013) introduce labor market frictions to the OLG model to analyze the effects of population aging and pension reforms in France. This model features full employment, and the frictions serve to endogenize the labor market exit decision of the elderly. However, it does not analyse the changes in the demographic structure, as population growth is constant over time. We follow Makarski et al. (2024) and introduce realistic demographic transition consistent with data, whereby there are fewer young individuals with every next birth cohort and longevity increases. This is reflected in the population growth and population structure, which align with the real (HP filtered) data.

Finally, we examine the existing literature on the changes occurring in the Polish labour

market. Poland is among the countries with the most rapid decline in the labour force due to a low fertility rate, which will result in a reduction of approximately 28% in the number of people of working age between 2015 and 2050 (Kielczewska and Lewandowski, 2017). One of the most significant potential consequences of this phenomenon is the reduction in the unemployment rate. Despite the recent influx of foreign workers, particularly Ukrainian nationals, into the Polish labour market, their presence has not resulted in any adverse effects on the unemployment rate (Duszczyk and Matuszczyk, 2018). In this paper, we therefore abstract from migration flows on the labour market and focus on the impact of population ageing on unemployment in Poland between 2000 and 2020. We allow this population ageing trend to filter into labour market decisions at the firm and worker level and observe its effects on the unemployment level. We then study whether economies differing by age composition exhibit differential cyclical properties, including the trade-offs between inflation and unemployment volatility.

3 Model

The model is identical to Makarski, Tyrowicz, and Radomska (2024). They construct a new Keynesian OLG model with sticky prices that features unemployment à la Mortensen and Pissarides (1994) to analyze the euro zone, whereas in this paper, we focus on Poland as an example of the fastest aging society in the European Union. We use this model to study the effects of demographic trends on the labor market, particularly unemployment. The model extends the approach of Bielecki et al. (2022) to account for labor market frictions. The key component of the OLG models (stochastic survival) is complemented with the key features of the DSGE models. Nominal rigidities are needed for monetary policy to have any role. Search and matching frictions are necessary to endogenously generate unemployment. In order for aggregate shocks to have the desirable effects on the economy, the model features investment adjustment costs and habit formation. The most important feature of this OLG model is that it allows for the incorporation of the real population structure and, therefore,

enables the analysis of the impact of demographic change on the labor market.

3.1 Households

In our model agents live for up to $J = 80$ period. Age is denoted by $j = 1, 2, \dots, J$ and one period equals one year, where age $j = 1$ corresponds to the age of 20 and J corresponding to 100 years in the data. Agents are subject to stochastic survival, where the conditional probability of an individual aged j is denoted by ω_j . Let $\mathcal{N}_{j,t}$ denotes the number of agents of age j in period t , with the total population given by

$$\mathcal{N}_t = \sum_{j=1}^J \mathcal{N}_{j,t} \quad (1)$$

with the growth rate of population denoted as $\nu_t = \mathcal{N}_t/\mathcal{N}_{t-1} - 1$.

Until retirement age $\bar{J} = 45$, in each period, a certain share of individuals in cohort j is unemployed $u_{j,t}$ and a certain share work $n_{j,t}$. Afterward for $j \geq \bar{J}$, they cannot work anymore. We assume that labor supply is determined by the labor market mechanism typical to the Mortensen-Pissaridies framework. Unemployed individuals receive unemployment benefits χ_t , and employed individuals receive real wage $w_{j,t}$ adjusted for age-specific productivity z_j . Wage income is taxed with the marginal tax rate τ_t . Households spend their income on consumption $c_{j,t}$ and assets $a_{j,t}$. Households can accumulate two types of assets: riskless bonds and risky assets. For more detailed description of asset accumulation see Makarski et al (2024).

Households receive returns from their financial asset holdings $a_{j,t}$ expressed in real terms. Investment funds manage assets, and return rates are age-specific $R_{j,t}^a/\pi_t$ with π_t denoting gross inflation rate and $R_{j,t}^a$ gross nominal return. The budget constraint is as follows:

$$c_{j,t} + a_{j,t} = (1 - \tau_t)w_{j,t}z_jn_{j,t} + \chi_{j,t}u_{j,t} + \frac{R_{j,t}^a}{\pi_t}a_{j-1,t-1} - T_t + beq_{j,t}, \quad (2)$$

where $beq_{j,t}$ denotes unintended bequests of birth cohort j at time t and T_t denotes lump-sum

taxes. The bequests are distributed evenly across cohorts that have less than 10 years to retirement, $j < \bar{J} - 10$.

Households draw utility from consumption. The following formula defines recursively utility function in period t of household at age j

$$\mathcal{U}_{j,t} = \frac{1}{1-\sigma} e^{\varepsilon_{c,t}} (c_{j,t} - \varrho \bar{c}_{j,t-1})^{1-\sigma} + \beta \omega^j \mathcal{U}_{j+1,t+1},$$

where β denotes the discount factor, ϱ denotes the degree of external habit formation, and $\varepsilon_{c,t}$ consumption preference shock that follows an AR(1) process with an autoregressive coefficient ρ_c and standard deviation of innovations σ_c . In our model we use aggregate variables expressed in per capita terms as Makarski et al (2024).

3.2 Firms

In our model, standard new Keynesian firm sector consists of monopolistically competitive intermediate goods firms and perfectly competitive final goods producers. Final good producers produce homogeneous final product y_t using the following CES production function $y_t = \left[\frac{1}{v} \int_0^v y_t(i)^{\frac{1}{\mu}} di \right]^{\mu}$, where $y_t(i)$ denotes differentiated intermediate good i and μ determines steady state markups of intermediate good firms. v denotes the number of intermediate good firms. Intermediate good producer i combine capital rented from households and labor to produce product variety $y_t(i)$ using the Cobb-Douglas technology

$$y_t(i) = e^{A_t} k_t(i)^\alpha \ell_t(i)^{1-\alpha} - \Phi \tag{3}$$

where Φ gives rise to the fixed cost. The number of firms v is fixed, and it is such that in the steady state, profits are zero. Firms set their prices according to the standard Calvo setting, with $1 - \theta$ denoting the Calvo probability of receiving the signal to re-optimize prices.

Capital goods are produced by combining the capital that did not depreciate from period

$t - 1$ and investment i_t with the following technology

$$(1 + \nu_{t+1})k_{t+1} = (1 - \delta)k_t + \left[1 - \frac{S_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t$$

where S_k determines the curvature of the investment adjustment cost. The nominal price of capital goods is denoted as $P_{k,t}$. For more detailed description of model features see Makarski, Tyrowicz and Radomska.

3.3 Labor market

In the labor market, unemployed of age j , $U_{j,t}$ meet vacancies, V_t , and new matches are created with the following technology

$$M_{j,t} = m_j(U_{j,t}, V_t) = e^{\varepsilon_{M,t}} \sigma_{j,m} (\mathcal{N}_{j,t}^{rel})^{1-\phi_j} U_{j,t}^{\phi_j} V_t^{1-\phi_j} \quad (4)$$

where $\varepsilon_{M,t}$ denotes the matching technology shock that follows an AR(1) process with standard deviation of innovations ρ_M and standard deviation of innovations σ_M .

We denote the probability of filling an open vacancy by a worker of age j is denoted by $q_{j,t} = M_{j,t}/V_t$, and the probability of finding a job by a worker of age j , $s_{j,t} = M_{j,t}/U_{j,t}$. We denote age specific labor market tightness as $\vartheta_{j,t} = \frac{V_t/\mathcal{N}_t}{u_{j,t}}$, where $u_{j,t} = \frac{U_{j,t}}{\mathcal{N}_{j,t}}$ denotes the unemployment rate. Thus

$$q_{j,t} = e^{\varepsilon_{M,t}} \sigma_{j,m} \mathcal{N}_{j,t}^{rel} \vartheta_{j,t}^{-\phi_j}$$

$$s_{j,t} = e^{\varepsilon_{M,t}} \sigma_{j,m} \vartheta_{j,t}^{1-\phi_j}$$

The share of employed $n_{j,t}$ and unemployed $u_{j,t}$ individuals from cohort j evolves according to the following formulas

$$n_{j,t} = (1 - \rho_{j-1})n_{j-1,t-1} + s_{j,t}u_{j,t} \quad (5)$$

$$u_{j,t} = 1 - n_{j-1,t-1}. \quad (6)$$

where ρ_j is an exogenously given age-specific probability of losing a job. Furthermore, for simplicity we assume that an exogenously given share of all workers entering labor market $n_{1,t}$ start as employed (due to timing in the model they can lose a job in the same period and be unemployed at the end of the period).

3.3.1 Job brokers

In our model, we delegate job brokering to competitive labor agencies. To facilitate the exposition, we divide job brokers into two brunches: the sale brunch and the hiring brunch. The first one aggregates the age-specific labor services into aggregate labor services sold to intermediate goods producers in the perfectly competitive market for the price of Ω_t . Job brokers maximize profits given by:

$$\Omega_t \ell_t - \sum_{\iota \in \{y,p,e\}} \Omega_{\iota,t} \ell_{\iota,t}, \quad (7)$$

where $\Omega_{y,t}, \Omega_{p,t}, \Omega_{e,t}$ denote the price of labor services of young, prime-age and elderly workers, respectively and the labor services in period t are given by the following formula

$$\ell_t = \left[\sum_{\iota \in \{y,p,e\}} (\ell_{\iota,t})^{\frac{\sigma_L-1}{\sigma_L}} \right]^{\frac{\sigma_L}{\sigma_L-1}}, \quad (8)$$

The second one (hiring brunch) does the search and matching in the labor makers, hires workers of different ages, and sells their work services to the sale brunch. In order to hire a worker, job brokers must post a vacancy V_t . The cost of posting a vacancy is $c(V_t)$ is given by

$$c(V_t) = \frac{\kappa}{2} \sum_{j=1}^{\bar{J}-1} q_{j,t}^2 V_t^2, \quad (9)$$

where κ is a constant. Job brokers choose $\{(\ell_{j,t})_{\iota \in \{y,p,e\}}, (N_{j,t})_{j \in \{1, \dots, \bar{J}-1\}}, V_t\}_{t=0}^{\infty}$ to maximize

the following objective function

$$E_0 \sum_{t=0}^{\infty} \left[\left(\prod_{s=0}^t \frac{\pi_s}{R_s} \right) \left(\sum_{\iota \in \{y,p,e\}} \Omega_{\iota,t} \ell_{\iota t} - \sum_{j=1}^{\bar{J}-1} w_{j,t} \ell_{j,t} - c(V_t) \right) \right], \quad (10)$$

subject to equations (8), $\ell_{j,t} = \frac{z^j N_{j,t}}{\mathcal{N}_t}$, and the law of motion for labor given by equation (5). For more detailed description we refer reader to Makarski, Tyrowicz and Radomska (2024).

3.3.2 Nash bargaining and wage setting

In our model, we assume wages are set according to staggered Nash wage bargaining. In each period, a job broker receives the signal to renegotiate wages with probability $1 - \theta_w$. The newly renegotiated nominal wage for cohort j is denoted as $\tilde{W}_{j,t}$, with $\tilde{w}_{j,t}$ denoting its real counterpart. Negotiations are described in detail below. If the firm does not receive the signal, it indexes the wages of its workers with inflation. In such a case, newly hired workers receive wages equal to those of individuals from their birth cohort. The newly hired workers of age $j = 1$ are an exception, as they receive the wage of labor market entrants from the previous period indexed to past inflation. Therefore, the average wage of cohort j in period t is given by

$$W_{j,t} = (1 - \theta_w) \tilde{W}_{j,t} + \theta_w \pi_{t-1} W_{j-1,t-1},$$

where $\pi_t = P_t/P_{t-1}$.

Firms renegotiating their wages in a period t do so with their entire workforce, including the new hires. Since workers of age j are homogeneous, they all receive the same wage. The details of the derivation of Nash bargaining and wage setting are presented in Makarski, Tyrowicz, Radomska (2024). Wages are either re-optimized or indexed to past inflation. Consequently, the average real wage for each cohort j in period t given by

$$w_{j,t} = (1 - \theta_w) \tilde{w}_{j,t} + \theta_w \frac{\pi_{t-1}}{\pi_t} w_{j,t-1}.$$

3.4 Monetary policy

We assume that monetary authorities conduct monetary policy according to the standard Taylor rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_t}{\bar{R}}\right)^{\gamma_R} \left(\left(\frac{\pi_t}{\bar{\pi}}\right)^{\gamma_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\gamma_y}\right)^{1-\gamma_R} \varepsilon_{R,t} \quad (11)$$

where \bar{x} denotes the steady state value of x . γ_R determines the extent of interest rate smoothing. γ_π and γ_y govern the responsiveness of the central bank to the deviation of, respectively, inflation and output from their steady state values. $\varepsilon_{R,t}$ denotes the iid monetary policy shock with standard deviation of innovations σ_R .

3.5 Government and the central bank

Government issues and services government debt given by b_t . Furthermore, it finances its expenditures, which are given by an exogenously given share of GDP $g_t = g_y y_t$, transfers to households T_t and unemployment benefits χ_t . The only source of revenue for the government budget is the tax on labor income τ_t . The government budget is given by the following formula

$$\frac{R_t}{\pi_t} b_t + g_t = (1 + \nu_{t+1}) b_{t+1} + \sum_j \tau_t w_{j,t} L_{j,t} + Div_t^{JB} + \sum_j \mathcal{N}_{j,t} T_t. \quad (12)$$

In the steady state, transfers to households are equal to zero $\bar{T} = 0$. Outside of it, they are given by the following fiscal rule¹

$$T_t - \bar{T} = \gamma_b (b_t - \bar{b})$$

3.6 Market clearing conditions

Market clearing in the final goods market requires

$$y_t = c_t + i_t + g_t + \bar{R} \Psi(cu) k_t \quad (13)$$

¹We use transfers to households to close the government budget as it is the least distortionary.

and the market clearing conditions for capital and labor can be written as

$$\frac{1}{v} \int_0^v k_t(i) di = k_t \quad (14)$$

$$\frac{1}{v} \int_0^v \ell_t(i) di = L_t \quad (15)$$

This allows us to write the aggregate production function as

$$y_t \Delta_t = A_t k_t^\alpha h_t^{1-\alpha} - \Phi \quad (16)$$

where price dispersion is given by

$$\Delta_t^{\frac{\mu}{1-\mu}} = \theta \left(\frac{\pi_t^\zeta}{\pi_t} \Delta_{t-1} \right)^{\frac{\mu}{1-\mu}} + (1-\theta) \tilde{p}_t^{\frac{\mu}{1-\mu}} \quad (17)$$

measures the price dispersion across intermediate goods. Full list of a standard set of market clearing conditions are presented in Appendix 3 of Makarski, Tyrowicz and Radomska (2024))

4 Calibration

We calibrate our economy to replicate the features of Poland in the 2015s.² We use annual data for calibration, as one period in our model corresponds to one year. For labor market characteristics, we rely on local data sources such as Central Statistical Office (the Labor Force Survey BAEL and The Demand for Labour Reports). For population structure, we refer to the World Population Prospects from the United Nations, and for economic characteristics, we utilize data from the National Bank of Poland and Eurostat.

²We calculate the features of the labor market in Poland as the mean from the years 2015 to 2019, excluding the year 2020 due to the pandemic, which altered the labor market characteristics.

4.1 Macroeconomic variables

Our calibration replicates the features of the Poland. Our steady state is calibrated to the period averaged over 2015 – 2019. During this period, the average yearly real interest rate amounted to 1.7%, which implies the discount factor $\beta = 0.98$. The capital depreciation rate δ of 0.066 allows match the investment rate of 18.6%. Likewise, we set capital share in output $\alpha = 0.22$ to match the capital to GDP ratio, which equals 2.68.

From OECD database we get the general government debt-to-GDP ratio which in years 2015 – 2019 was on average equal to 68.6% of GDP.³ The curvature of the investment adjustment cost is set at 4.

Following, Makarski, Tyrowicz and Radomska (2024) and Bielecki et al. (2022) we set habit persistence parameter ρ at 0.75⁴, price stickiness θ at 0.75⁴, and weight of past inflation in the indexation rule ζ_π at 0.25. We set the wage stickiness at $\theta_w = 0.85$ ⁴. Finally, we choose the responsiveness of employment of labor market entrants to GDP γ_n at 2. The match between the volatility of these two measures is consistent with the data. We set the parameter $\Psi = \Psi_2/\Psi_1 = \frac{0.85}{1-0.85}$.

4.2 Population heterogeneity

In our model, the agents work until $\bar{J} - 1 = 44$. We distinguish three age cohorts: young, prime-age, and elderly. The agent at the labor market is young for 10 periods $j \in \{1, \dots, 10\}$, prime-age for 25 periods $j \in \{11, \dots, 35\}$ and elderly for 9 periods $j \in \{36, \dots, 44\}$. To obtain the age-specific productivity profiles of agents (z_j), we use the SES (The Structure of Earnings Survey) from Eurostat for Poland and apply the Deaton decomposition. Figure 4 presents the smoothed labour productivity data, which was modelled using a third-order polynomial. The asset structure of the population is adopted from the Polish Central Bank

³From OECD: “The general government debt-to-GDP ratio measures the gross debt of the general government as a percentage of GDP. It is a key indicator for the sustainability of government finance. Debt is calculated as the sum of the following liability categories (as applicable): currency and deposits; debt securities, loans; insurance, pensions and standardised guarantee schemes, and other accounts payable. Changes in government debt over time primarily reflect the impact of past government deficits.”

Report (based on the results of 2016 survey) and Grejcz and Żółkiewski (2017) and replicates directly the features described in this study. The asset structure is smoothed by incorporating the quadratic interpolation for the asset structure of people aged 20-99, as illustrated in Figure 5.

4.3 Labor market parameters

In our model, the household agent can work full-time or not at all (is unemployed). We calibrate labor market flows parameters, such as the outside option (the replacement ratio of the unemployment benefit relative to wages), the find rate, the separation rate, and the vacancy posting cost. The parameters are calculated as an average for the period 2015-2019.

We rely on individual-level data from the Labor Force Survey (BAEL) to derive job-finding and separation rates. Survey participants report their labor market status in the current quarter and the preceding year. This information allows us to compute the intensity of flows from unemployment to employment (job-finding rate) and from employment to unemployment (separation rate). Our model encompasses two states: employment and unemployment. Figure 6 depict labor market flows (job finding rate and separation rate) for three age groups (young, prime-age, and elderly). Figure 7 illustrates the existence of notable discrepancies in the unemployment rate across three age groups (young, prime-age, and elderly). The unemployment rate exhibited a downward trend over time for the period spanning 2000 to 2019. Furthermore, the disparity in the unemployment rate between younger and other workers has diminished from approximately 15 percentage points (the beginning of the 2000s) to approximately 5 percentage points (before the 2020 pandemic).

We deploy individual-level data to obtain the finding and separation rates because we need values separately for three cohorts: young, prime-age, and elderly. In our model, agents either work ($N_{j,t}$) or are unemployed ($U_{j,t}$). We calculate the job finding and separation rate as follows:

$$s_{j,t} = \frac{UN_{j,t}}{U_{j,t-1}} \text{ and } \rho_{j,t} = \frac{NU_{j,t}}{N_{j,t-1}}$$

where $U_{j,t-1}$ ($N_{j,t-1}$) is the number of unemployed (employed) from j -cohort in period $t - 1$, $U_{j,t}$ ($N_{j,t}$) is the number of unemployed (employed) that were in the previous period employed (unemployed) from j -cohort in period t . In other words, $NU_{j,t}$ and $UN_{j,t}$ measure the flows to and from unemployment.

We report the evolution of the find rate $s_{j,t}$ and separation rates $\rho_{j,t}$ in Figure 6. Our main calibration takes the values averaged for 2015 – 2019. For our 2015’s calibration, we use the calculated values for separation rates $\rho_y = 0.03$, $\rho_p = 0.02$, $\rho_o = 0.014$ for young, prime-age and elderly cohorts, respectively. We calibrate the scaling parameter of the matching $\sigma_{j,t}$ function to 0.9, 0.75, and 0.59 for young, prime-age, and elderly cohorts, respectively, to match the empirically observed job-finding rates reported in Figure 6.

The average unemployment benefit in Poland in years 2001-2005 was 50.8% of the previous wage. In years 2015 – 2019 it was 33.6% of the previous wage.⁴ We set the parameter $\chi = 0.44$, implying that the unemployed receive 34.3% of the average wage. Following the Makarski, Tyrowicz and Radomska (2024) we set the elasticity of the matching function equal to $\phi = 0.72$. Following the Hosios condition, we set the workers’ bargaining power equal to the elasticity of the matching function, namely $\eta = 0.72$.

We calculate the labor market tightness $\vartheta = V/U$, where V denotes the number of posted vacancies and the U denotes the number of unemployed. We use the data regarding posted vacancies and unemployment from the Polish Central Statistical Office (The Demand for Labour Reports for years 2005-2020). We obtain $\vartheta = 0.1$ and to match it, we calibrate the cost of posting the vacancy $\kappa = 32.1$.

To separate the effect of the changes on the labor market from the demographic change we make a deterministic simulation. First, we calibrate the labor market parameters to 2000s (mean from 2001-2005)⁵ and set the values for parameters for the beginning period of

⁴From OECD database (BENUNEMPPC.1YEAR.PC_PWI): This indicator measures the proportion of previous in-work household income maintained after 12 months of unemployment.

⁵This is the oldest period we can consider because before 2001, the Labor Force Survey (BAEL) did not include questions about the last year’s employment status. Therefore, it is impossible to evaluate labor market flows from/to employment/unemployment, making it impossible to calculate the separation and job finding rates.

transition analysis. Then we allow for immediate update within one period in 2001 of labor market parameters: the number of employed young workers entering the labor market N_{first} , cost of posting the vacancy κ_t , the scaling parameter of matching function $\sigma_{j,t}$ and separation rates $\rho_{j,t}$. All other market parameters remain constant. A more detailed description of the deterministic simulation is presented in Section 5.

4.4 Demographics

We use the World Population Prospects published by the United Nations to obtain past and projected estimates of population structure, mortality, and population growth for Poland (from 1950 to 2100). Our model requires age-dependent mortality risk and population structure; hence, we use 'Life Tables - Single Ages for both sexes' (estimates and medium projection) and 'Population by Single Age - Both Sexes'.⁶ We smooth the population similar to Bielecki et al. (2020) and Makarski et al. (2024) by using the Hodrick-Prescott filter with a smoothing parameter of 10,000.⁷ During the calibration process for 2015s, we fixed the population, mortality and population growth rate at the 2015 level.

4.5 Model match to the data

In Table 2, we report the match between the target statistics in the model and in the data. We calibrate the model using the target statistics for two steady states: 2000s and 2015s. Figure 8 shows how the model fits the data for the total unemployment rate and for unemployment rate for three age groups: young, prime and elderly. Our model quite well replicate the trends of unemployment (Figure 8) and job finding rates (Figure 9) for three groups (young, prime-age and elderly workers), taking into account that we only calibrated the model at two points in time: for the 2000s and 2015s, allowing for an immediate change

⁶available on the webpage of the United Nations: <https://population.un.org/wpp>.

⁷More precisely, we recalculate the population based on the mortality and population of 20-year-olds, considering 2010 as the main reference year for population structure. This is a standard procedure, particularly for analyzing demographic changes on the transition path. The model does not converge with the 'raw' data. Therefore, a large smoothing parameter allows us to capture secular trends in fertility rates rather than reflecting post-war baby booms and their echoes. For additional details, refer to Bielecki et al. (2020).

of parameters in one period.⁸ We passed all the macroeconomic and labor market variables through the Hodrick-Prescott filter with smoothing parameter 100 to separate fluctuations and trends, what is standard practice in macromodelling (Bielecki et al. (2020) and Makarski et al. (2024)).⁹

4.6 Stochastic shocks

The economy is driven by eight stochastic shocks: namely, productivity, preferences, government expenditure, monetary policy, investment and unemployment for each age group (youth, prime-age and elderly). To estimate the parameters regarding these shocks we use procedure from Makarski, Tyrowicz and Radomska (2024), which is based on the simulated method of moments used in Ruge-Murcia (2012). First, we assume that productivity, preferences, government expenditure, investment and unemployment for each age group shocks follow first-order autoregressive processes, and the monetary policy shock is white noise. Than parameters governing these shocks are estimated using the simulated method of moments which minimize the distance between the selected moments from the data and their model-based counterparts (see Makarski, Tyrowicz and Radomska (2024) for more details).

Table 3 presents the parameters of shocks obtained in this procedure and table 4 shows the model data fitted to selected moments in the Polish economy. Our model reproduces the main features of the Polish business cycle, in particular the volatility of unemployment by age groups. The volatility of consumption and investment is slightly higher than in the data, while the volatility of the interest rate and inflation is slightly lower than in the data. In the case of autoregressions, they are generally higher in the model than in the data, except for the interest rate, consumption and output, which is lower in the model. Finally, our model matches the correlations with output quite well for most variables, apart from the interest

⁸The common macroeconomic practice is to change the parameter values between two steady states in one period and then simulate the path transitions between the two steady states, which we have used in this exercise.

⁹For example, we obtain the detrended unemployment rate by calculating the $U_{detrended} = e^{\frac{HP(100 \log U, 100)}{100}}$, where U is the unemployment time series and $HP(100 \log U, 100)$ is the Hodrick-Prescott filter with smoothing parameter 100 applied to logarithm of total unemployment rate. This procedure was applied to all data used for calibration.

rate.

5 Deterministic simulations results

In this section we discuss the results of the deterministic simulations in order to quantify the impact of labour market and demographic changes on unemployment levels. To this end, we carry out a counterfactual analysis, taking into account the changes in the labour market and in the population structure between 2000 and 2020, and decompose the impact of these two channels on unemployment. The results of our simulations indicate that the reduction in unemployment in Poland between 2000 and 2020 is mainly due to shifts in the labour market, with less than 20% of the reduction in unemployment due to demographic changes.

We also employ our model to project the anticipated level of unemployment in Poland up to the year 2040. It is assumed that the labour market will remain unaltered, with the values of the all parameters being taken from the 2015s steady state. The impact of demographic change, specifically population ageing, on the level of unemployment is then assessed.

5.1 Macroeconomic scenarios

Following Makarski et al (2024), we conduct three demographic scenarios. In the first scenario, we isolate the effects of demographics by decomposing the changes into composition effects and direct changes in unemployment rates. We keep the Polish labour market parameters unchanged, equal to the calibration from the 2000s, and allow the population structure to change as it is in the data (and demographic projections) between 2000 and 2040 (the change in population growth n_t and population structure N_{jt}). We decompose the impact of demographic change on the total unemployment u_t for years 2000 – 2019 into two effects: the composition effect and the behavioral effect. The total unemployment rate can be expressed as a weighted sum of the unemployment of three age groups as:

$$u_t = \omega_{y,t}u_{y,t} + \omega_{p,t}u_{p,t} + \omega_{o,t}u_{o,t},$$

where y , p , and o denote young, prime age and elderly workers, respectively. In this notation, u denotes unemployment and ω denotes the share of a given age group in the working population. Denote u_t the simulated unemployment rate in a given scenario and u_0 the unemployment rate in the base scenario. Then, we can rewrite the above equation as a shift-share decomposition to the two effects: a composition effect and a behavioral effect:

$$u_t - u_0 = \underbrace{(\omega_{y,t} - \omega_{y,0})u_{y,t} + (\omega_{p,t} - \omega_{p,0})u_{p,t} + (\omega_{o,t} - \omega_{o,0})u_{o,t}}_{\text{composition}} + \underbrace{\omega_{y,0}(u_{y,t} - u_{y,0}) + \omega_{p,0}(u_{p,t} - u_{p,0}) + \omega_{o,0}(u_{o,t} - u_{o,0})}_{\text{behavioral}}.$$

In the second scenario, we keep the population unchanged (structure from the 2000s calibration) and allow for the changes in the labor market parameters. We change the labor market parameters $\rho_{i,t}$ (separation rate), $\sigma_{i,t}$ (scaling parameter in the matching function), $n_{1,t}$ (number of employed of the entering cohort) and κ_t (cost of posting the vacancy) according to the values of the calibrated parameters for 2000s and 2015s.

In the third scenario we allow simultaneously for both changes: demographic and labor market change, which allows us to show the joint effect of both demographic changes and the changes in the functioning of the labor market. These three scenarios allow us to decompose the impact of demographic change and the labor market change on the total unemployment u_t .

Figure 10 shows the results of the deterministic simulations for the three scenarios for Poland. The upper left panel shows the decomposition of the aggregate unemployment, while the remaining three panels show the decompositions for young, prime-age and elderly workers. The black solid line represents the joint effects of demographics and labor market flows, while the bars depict the decompositions for a demographic scenario and labor market flow. Because our model is not linear, the sum of three bars (composition and behavioral effects of demographic changes and labor market changes) does not add up to the solid line.

The most significant reduction in the overall unemployment rate in Poland, amounting to approximately 12 percentage points, can be attributed to shifts in labour market dynamics. Across all three age groups, we observe a notable decline in unemployment, largely attributable to changes in the labour market. The demographic transition accounts for a modest increase of slightly more than 2.5 percentage points. The composition effect contributes approximately 0.7 percentage point to the decline in unemployment in Poland, while the behavioural effect accounts for approximately 1.8 percentage point. The demographic effect is observed among all age group workers, with slightly higher effects observed for those in their prime working years.

5.2 Macroeconomic projections

We use our model to project the expected level of unemployment in Poland up to the year 2040. We have assumed that the labour market will remain unchanged, with the values of all parameters taken from the steady state in 2015. To be more precise, we keep the values of $\rho_{i,t}$ (separation rate), $\sigma_{i,t}$ (scaling parameter in the matching function), $n_{1,t}$ (number of employed of the entering cohort) and κ_t (cost of posting the vacancy) unchanged and equal to its 2015s values. The impact of demographic change, specifically population ageing, on the level of unemployment is then assessed. Due to changes in the population structure and the long-term convergence to its steady state, the job finding rate also changes, as illustrated in the Figure 12.

The projected unemployment rates for young, prime-aged, and elderly workers, as well as for the overall population, are illustrated in Figure 11. In the absence of any changes in the labour market, the unemployment rate would decline for all age groups, with the most pronounced decrease observed for the youngest age group (approximately 1.2 percentage points). The projected decrease in unemployment for the prime-age cohort is 0.9 percentage points, while the projected decrease for the elderly cohort is 0.8 percentage points. The total unemployment rate is projected to decrease by over 1 percentage point, with the composition effect accounting for approximately 0.3 percentage point and the behavioural effect account-

ing for approximately 0.9 percentage point. Both the behavioural effect and composition effect are increasing over time, indicating that the decrease in unemployment in Poland will continue for the next 20 years due to the ageing of the population.

6 Stochastic simulations results

The aim of this section is to analyse the implications of demographic developments for the conduct of monetary policy. To this end, stochastic simulations of the economy are carried out around the steady state with fixed labour market parameters from 2015 and different population structures. In order to identify the impact of demographic change on monetary policy, the stochastic behaviour of the economy is compared for populations in 2015 and 2000.

6.1 Impulse response function to monetary shock

The Figure 14 illustrates the impulse response function and thus the impact of an ageing workforce on the monetary transmission mechanism. The objective of this exercise is to examine the impact of an interest rate shock on economic outcomes. To achieve this objective, three scenarios are presented. The first scenario concerns an economy calibrated to 2015 data (both population structure and labour market parameters). This scenario corresponds to the main calibration model. For comparison, we also present the results of a simulation in which the economy is subjected to a monetary policy shock while all market and labour market parameters remain constant and the population is taken from the year 2000 and from 2040.

The simulations illustrate the typical response of the economy to a shock, namely an increase in interest rates. A comparable decline in output, consumption, and inflation is observed in all three scenarios. In all three scenarios, unemployment increases across all age groups. However, in the scenario from main calibration where the population is older (from 2015), the rate of return to the long-run level of unemployment is faster than in the scenario where the population is younger (from 2000). These differences are particularly pronounced

when the population structure is taken from the 2040 projection.

In order to identify the source of this discrepancy, a comparison is made between the responses of the youth, prime-age and elderly unemployment rates. While the oldest population (red dotted line) from 2040 shows the highest response of the unemployment rate of the three groups to the monetary shock, the return to the long-term level of unemployment is the fastest compared to the scenario with the population from 2015 and 2000. Our results show that the response of the total unemployment rate is less pronounced than that of the three age groups (Figure 14). It can therefore be concluded that the behavioural effect is the main source of this difference.

6.2 Monetary policy frontier

The previous section suggests that demographic shifts in the labour market have the potential to affect the trade-offs faced by monetary policy. To examine these effects, we analyse how demographic trends affect the monetary policy frontier, following the methodology proposed by Makarski et al. (2024). Central banks aim to achieve a number of objectives, including controlling inflation, stabilising employment and maintaining financial stability. However, these objectives may conflict with each other. For example, implementing a tighter monetary policy (by raising interest rates) to control inflation can lead to a slowdown in economic growth and an increase in unemployment. In this exercise, we present a visual representation of the optimal combinations of inflation and output, as well as the variability of inflation and unemployment, and show that demographic change has an impact on the optimal policy recommendations.

The monetary policy in our economy follows the standard Taylor rule, as represented by (11), which responds to deviations of output and inflation from their respective steady-state values. In order to obtain the monetary policy frontier, we minimize the standard central bank loss function consisting of inflation and unemployment volatility with the full range of weights on the two target values (see Makarski et al.(2024) for a detailed description of the methodology).

Figure 15 illustrates the policy frontiers generated from the model calibrated to the 2015s with three different population structures, younger from 2000, older from 2015 and from the 2040 projection. The main result concerns the sacrifice ratio. It shows how much GDP volatility must be given up to reduce inflation volatility by one percentage point, which is represented by the absolute value of the slope of the monetary policy frontier. Our simulations show that the sacrifice ratio increases with an older population (from 2015) compared to a younger one (from 2000). We observe the highest sacrifice ratio for the population from the projection (2040), which is the oldest population in all three cases. In other words, reducing the standard deviation of inflation by one percentage point costs less in terms of the standard deviation of GDP with an older population. Intuitively, this means that the response of the GDP is weaker as the labour force ages. Monetary policy therefore needs to generate larger movements in unemployment, and hence output, in order to stabilise inflation.

Furthermore, we examine the extent to which an increase in unemployment volatility would be required to reduce inflation volatility by one percentage point. Figure 16 shows that in the case of the unemployment rate, the sacrifice ratio also increases, and this change is more pronounced. This observation may have significant implications for the conduct of monetary policy. One of the key challenges of monetary policy is the need to achieve a delicate balance between two potential adverse outcomes: inflation and unemployment. With a higher sacrifice ratio and a reduced responsiveness of unemployment to monetary policy, the optimal monetary policy should be more inflation-oriented with the older workforce. Consequently, central banks may adopt a more hawkish stance as societies age.

7 Conclusions

Poland presents a fascinating case for studying unemployment dynamics due to its remarkable economic and demographic transformation since 2000. Over this period, unemployment has significantly declined, accompanied by robust economic growth. At the same time, Poland

has become one of Europe's fastest-aging societies, driven by its exceptionally low fertility rate. Our paper contribute to the literature in several dimension. First, using the data, we present the labor market characteristics for Poland for years 2000-2020: the job finding rates, separation rates and unemployment rates for three age groups (youth, prime-age and elderly workers). Moreover using the Deaton decomposition we estimate the labor productivity for Polish workers regarding for age. Second, we conduct the counterfactual analysis to decompose the unemployment rate since 2000. Our analysis shows that more than 80% of the decline in the unemployment rate between 2000 and 2020 can be attributed to changes in the labour market, while less than 20% can be explained by demographic changes, such as the ageing of the population.

Third, we present the projection of the unemployment rate in Poland for the next 20 years (between 2020 and 2040) for all three age groups. Assuming no changes in the labour market, we estimate that the overall unemployment rate in Poland should decrease by more than 1 percentage point by 2040 due to the ageing of the population. The largest decrease is observed for young workers (1.3 percentage points), while for prime-age and older workers the decrease is less than 1 percentage point.

Fourth, we present the implications of demographic trends for monetary policy. In particular, the model suggests that the sacrifice ratio is lower in older working populations, suggesting that monetary policy in ageing societies should prioritise stabilising inflation over reducing unemployment. This reflects a more favourable trade-off between inflation and unemployment under a restrictive policy stance.

To investigate these phenomena, we use a novel New Keynesian overlapping generations (OLG) model with labour market frictions. Our model successfully replicates aggregate unemployment levels and trends across age groups in two steady states: 2000s and 2015s, and replicates quite well the changes in the transition path of both the unemployment rate and the job finding rate. We use the moment-matching procedure to improve the predictive accuracy, especially with respect to the policy frontier and the inflation-unemployment trade-off.

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Tables and figures

Table 1: Calibrated structural parameters for Poland

Parameter	Value		Description
	2000s	2015s	
A. Households			
β	0.98	0.98	Discount factor
ϱ	0.75 ⁴	0.75 ⁴	Habit persistence
B. Firms			
δ	0.066	0.066	Capital depreciation rate
α	0.22	0.22	Capital share in output
S_K	4	4	Investment adjustment cost curvature
μ	1.2	1.2	Steady state product markup
θ	0.75 ⁴	0.75 ⁴	Calvo probability (prices)*
ζ_π	0.25	0.25	Weight of past inflation in prices indexation*
Φ	0.04	0.04	Intermediate goods producers fixed cost
C. Labor market			
κ	23.5	32.1	cost of posting the vacancy
N_{first}	0.61	0.83	number of employed young entering the market
ρ_{young}	0.07	0.03	separation rate for young
ρ_{prime}	0.04	0.02	separation rate for prime
ρ_{old}	0.02	0.014	separation rate for old
σ_{young}	0.81	0.9	scaling parameter in the matching function
σ_{prime}	0.55	0.75	scaling parameter in the matching function
$\sigma_{elderly}$	0.4	0.59	scaling parameter in the matching function
ϕ_j	0.72	0.72	elasticity of matching function
η	0.72	0.72	parameter in the Nash bargaining process
θ_w	0.85 ⁴	0.85 ⁴	nominal wage stickiness
χ	0.44	0.44	unemployment benefit
σ_L	5	5	substitutability of workers
γ_n	2	2	responsiveness of labor market entrants employment to GDP*
D. Government and central bank			
π	1.02	1.02	Steady state inflation
γ_R	0.8 ⁴	0.8 ⁴	interest rate smoothing
γ_π	1.97	1.97	reaction to inflation
γ_y	0.41	0.41	reaction to GDP growth
γ_b	0.42	0.42	fiscal rule parameter
Ψ	$\frac{0.85}{1-0.85}$	$\frac{0.85}{1-0.85}$	capacity utilization costs

*Note: Parameters θ , ζ_π , and γ_n are set to 0 in the deterministic simulations.

Table 2: Target statistics in the data and the model for Poland

variable	2000s		2015s		description
	model	data	model	data	
r	2%	-	1.7%	1.7%	real interest rate
b^g/y	68.6%	-	68.6%	68.6%	steady state government debt to GDP ratio
$\frac{i}{y}$	19%	-	18.5%	18.6%	investment rate
$\frac{k}{y}$	2.6	-	2.7	2.7	capital to GDP ratio
u_{young}	26.8%	26.7%	9.1%	9.1%	unemployment rate for young
$u_{prime\ age}$	14.6%	14.6%	4.1%	4%	unemployment rate for prime age individuals
$u_{elderly}$	11.7%	11.6%	3.8%	3.8%	unemployment rate for elderly
u_{total}	17.3%	17.6%	5.2%	5.6%	total unemployment rate
s_{young}	30.6%	30.8%	48.2%	48.9%	job finding rate for young
$s_{prime\ age}$	24.5%	24.4%	48.3%	47.3%	job finding rate for prime age individuals
$s_{elderly}$	18.9%	18.3%	38.9%	38.5%	job finding rate for elderly
ϑ	0.1	-	0.2	0.1	market tightness

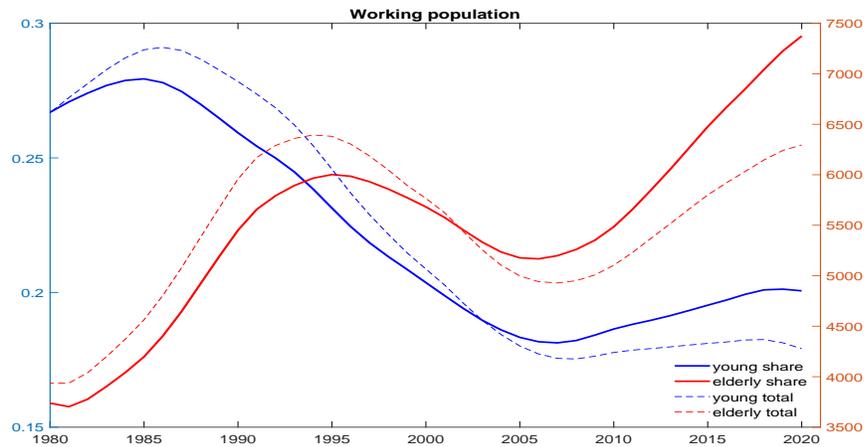
Table 3: Stochastic shocks obtained in moment matching procedure for the Polish economy

Parameter	Value	Description
A. Persistence		
ρ_A	0.35722	Productivity shock - autocorrelation
ρ_c	0.999	Preference shock - autocorrelation
ρ_g	0.56557	Gov. expenditure shock - autocorrelation
ρ_I	0.76796	Investment shock - autocorrelation
ρ_Y	0.77977	Youth unemployment shock - autocorrelation
ρ_P	0.905	Prime-age unemployment shock - autocorrelation
ρ_E	0.3	Elderly unemployment shock - autocorrelation
B. Standard deviations		
σ_A	$1.3493e^{-5}$	Productivity shock - standard deviation
σ_c	0.15141	Preference shock - standard deviation
σ_g	0.0036154	Gov. expenditure shock - standard deviation
σ_R	0.032939	Monetary shock - standard deviation
σ_I	$1.1283e^{-5}$	Investment shock - standard deviation
σ_Y	$8.7255e^{-5}$	Youth unemployment shock - standard deviation
σ_P	0.066476	Prime-age unemployment shock - standard deviation
σ_E	0.25362	Elderly unemployment shock - standard deviation

Table 4: Selected moments in the Polish economy

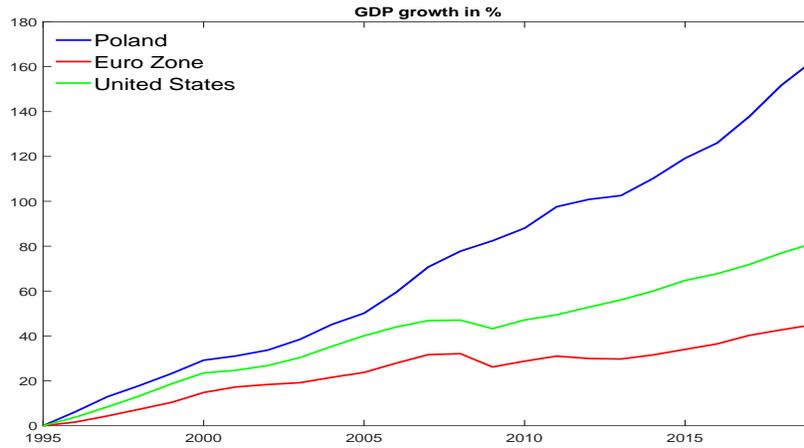
Variable	Standard Deviations		Correlation with output		Autocorrelation	
	data	model	data	model	data	model
variables used in moment matching						
output	1.8	1.8	1	1	0.67	0.56
consumption	2.1	2.4	0.84	0.67	0.73	0.50
interest rate	2.7	2.5	-0.0	-0.93	0.84	0.34
inflation	1.7	1.1	0.6	0.27	0.49	0.76
investment	7.9	8.5	0.75	0.53	0.59	0.99
unemployment	22.2	16.7	-0.85	-0.56	0.78	0.97
unemployment young	17.4	17.4	-0.8	-0.72	0.74	0.95
unemployment prime	20.0	20.0	-0.8	-0.37	0.71	0.98
unemployment old	18.4	18.4	-0.78	-0.35	0.71	0.91

Figure 1: Working population structure (young vs. old) in Poland



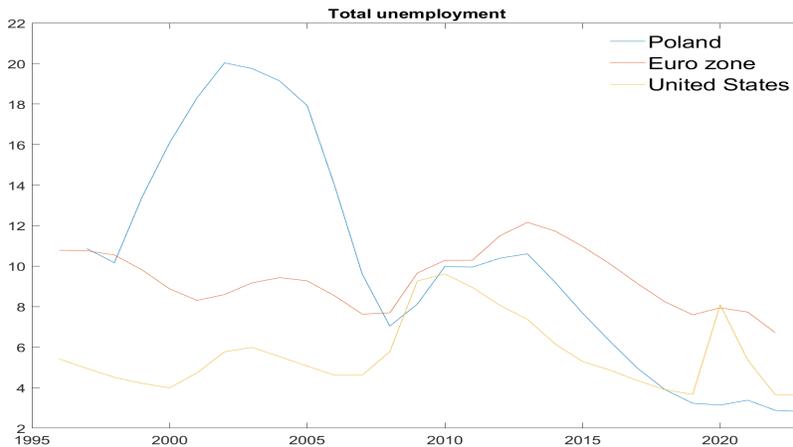
Note: the data come from the United Nations World Population Prospects 2019, which covers the years 1950-2020 (actual data). The shares of the young and old populations are calculated by dividing the sum of the young age population (aged between 20-29) and the sum of the old population (aged between 55-64) over all people in the working age (aged between 20-64).

Figure 2: GDP growth



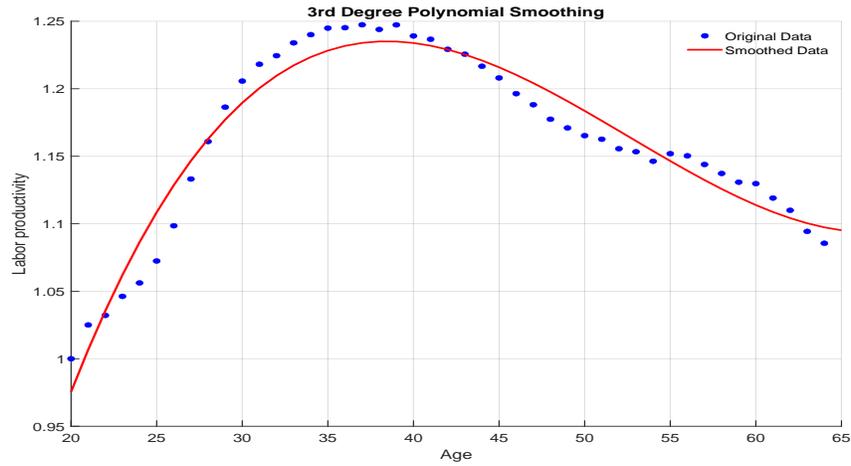
Note: the data come from Federal Reserve Economic Data: CLVMNACSCAB1GQPL (Poland), CLVMEURSCAB1GQEA19 (Euro Zone) and GDPC1 (United States). The historical time series are expressed in the local currency (PLN, EUR, USD, respectively), but similar trends are observed for time series expressed in USD. The year 1995 was selected as the base year.

Figure 3: Unemployment rate



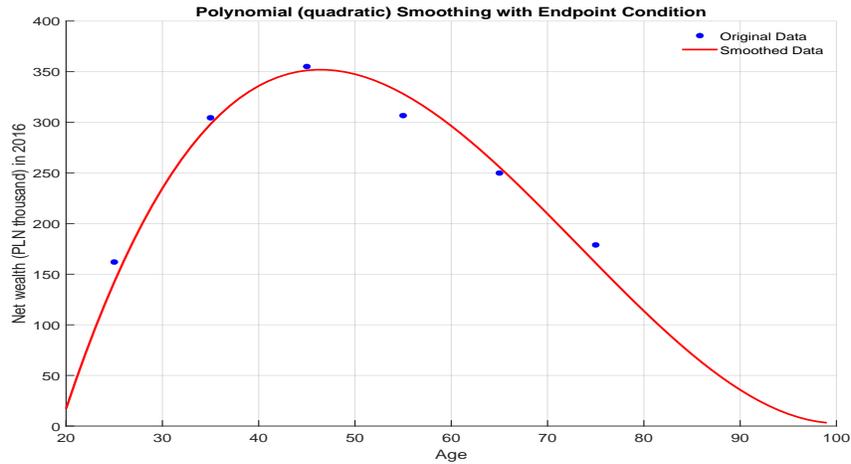
Note: the data come from Federal Reserve Economic Data: LRHUTTTTPLA156N (Poland), LRHUTTTTEZA156N (Euro Zone) and LRHUTTTTUSA156N (United States).

Figure 4: Labor productivity



Note: the original data come from own Deaton decomposition

Figure 5: Asset structure for Poles



Note: The original data were sourced from the Polish Central Bank Report and represent the beginning of the age periods: 25-34; 35-44; 45-54; 55-64; 65-74; 75+. These age periods correspond to the age groups for which net wealth was reported.

Figure 6: Labor market flows in Poland: job find rate, $s_{j,t}$ (left) and separation rate, $\rho_{j,t}$, (right)

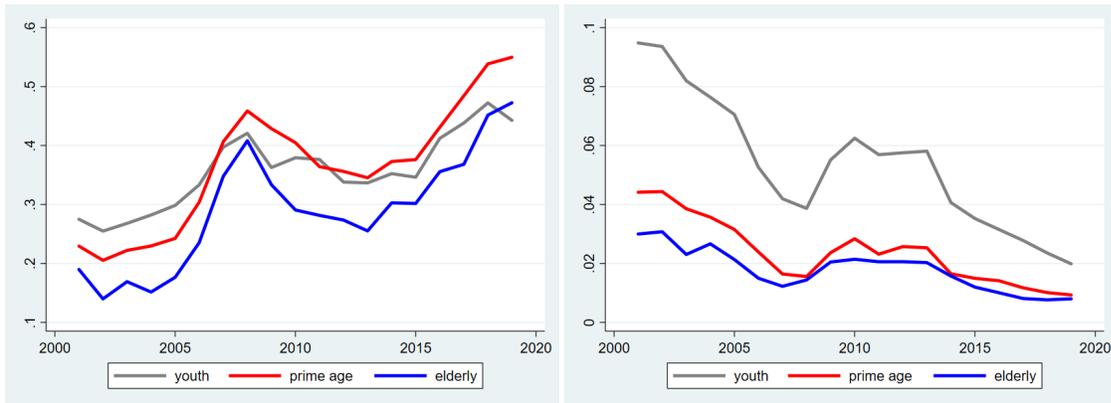


Figure 7: Unemployment rate for Poland

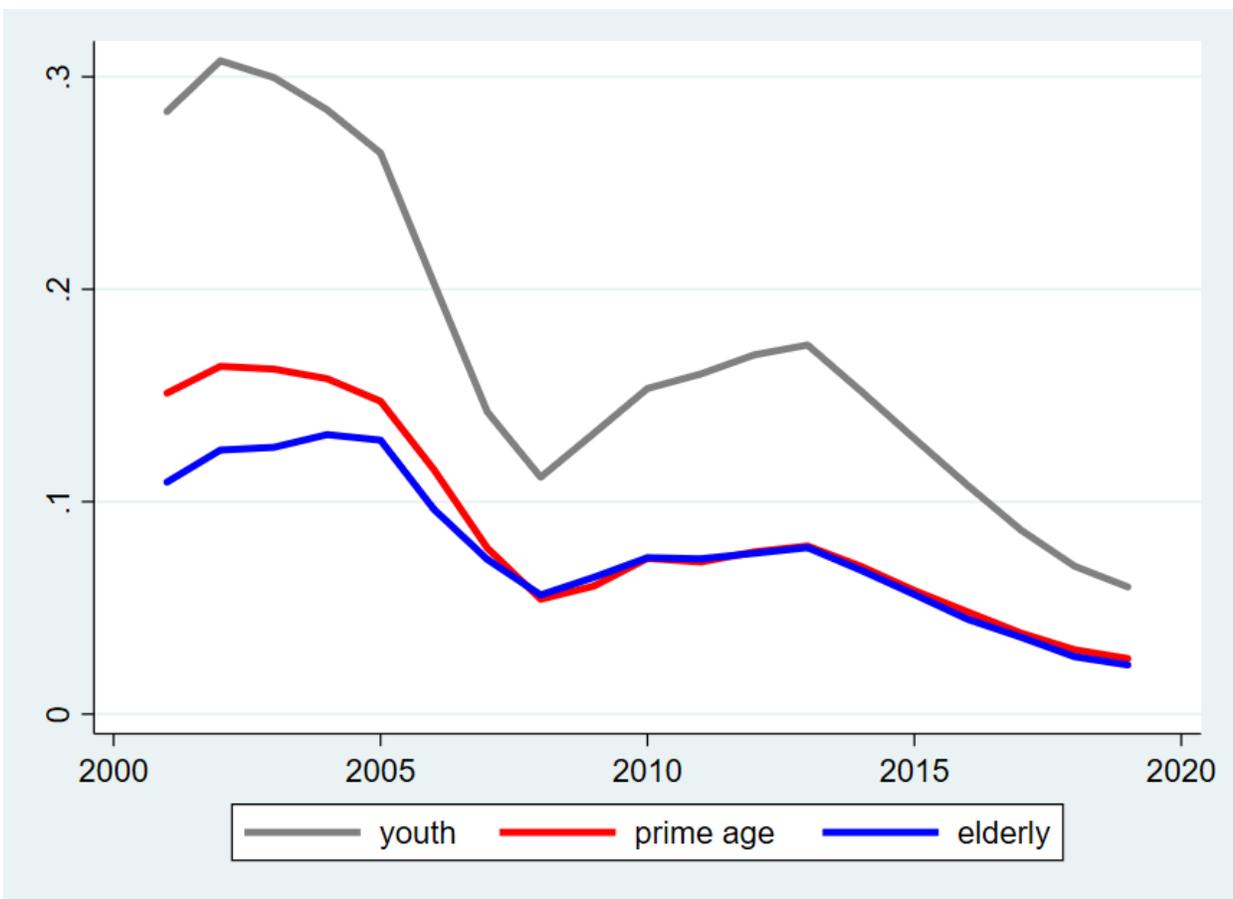


Figure 8: Unemployment rate rate for all age group, young, prime and elderly workers: model vs data and HP trend for Poland

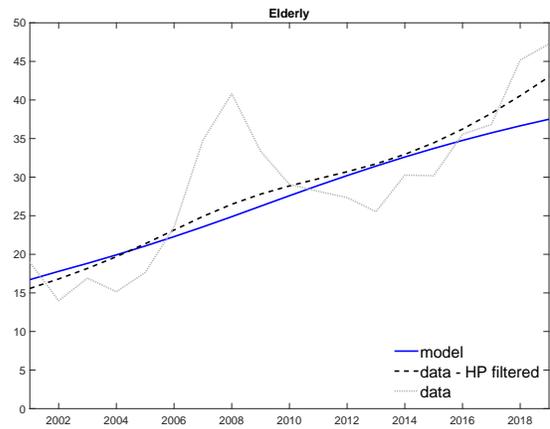
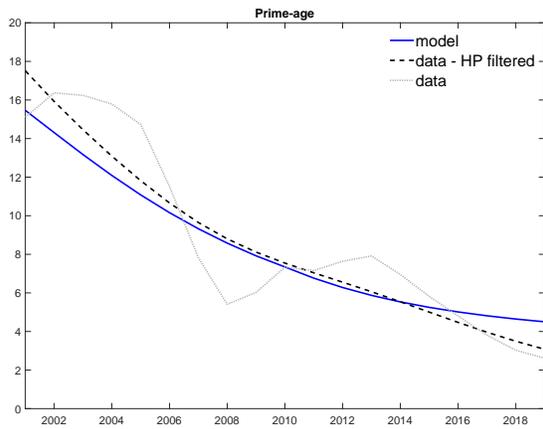
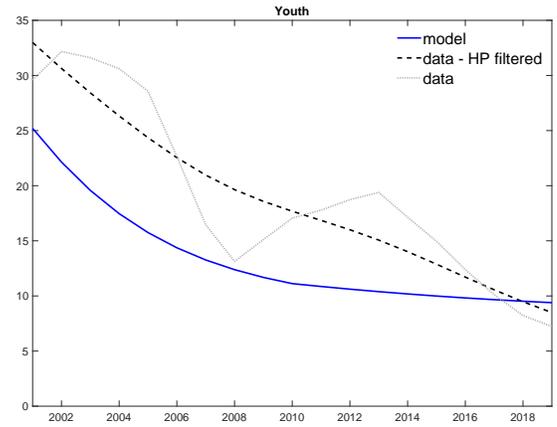
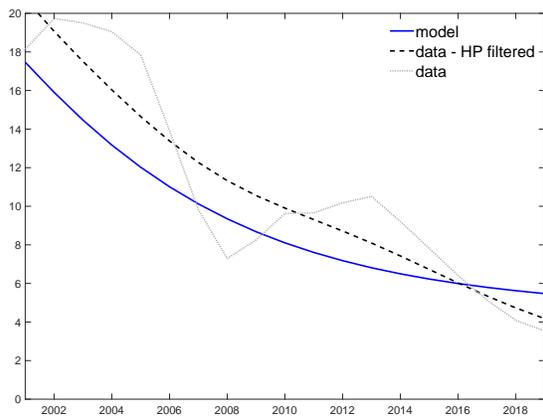


Figure 9: Job find rate rate for young, prime and elderly workers: model vs data and HP trend for Poland

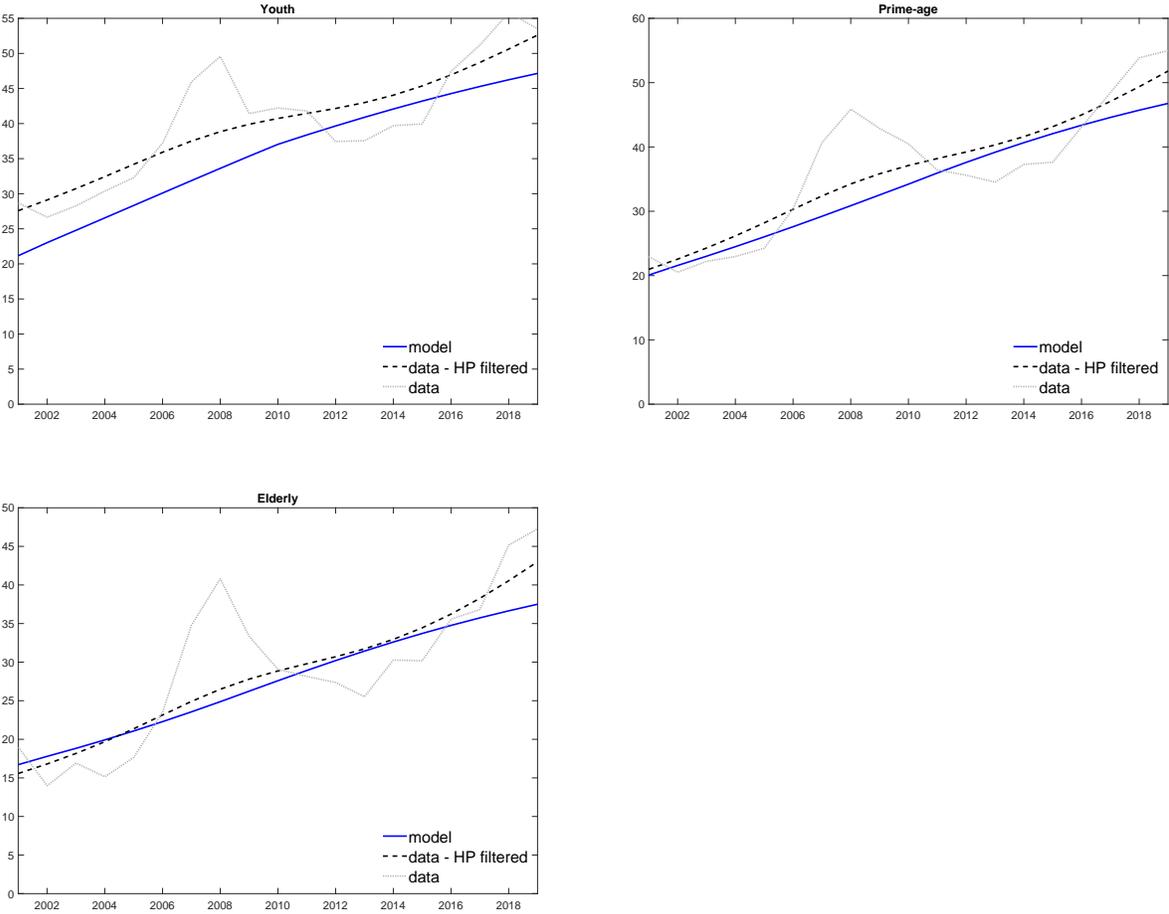


Figure 10: Decomposition of unemployment change for Poland

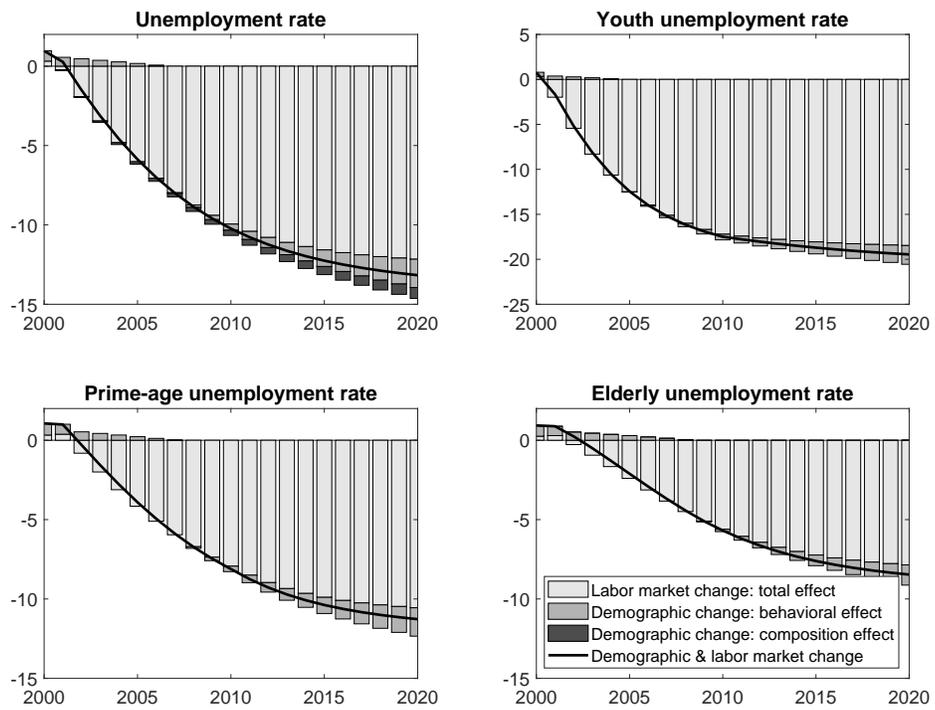


Figure 11: Projection of unemployment rate rate for all age group, young, prime and elderly workers: model vs data and HP trend for Poland

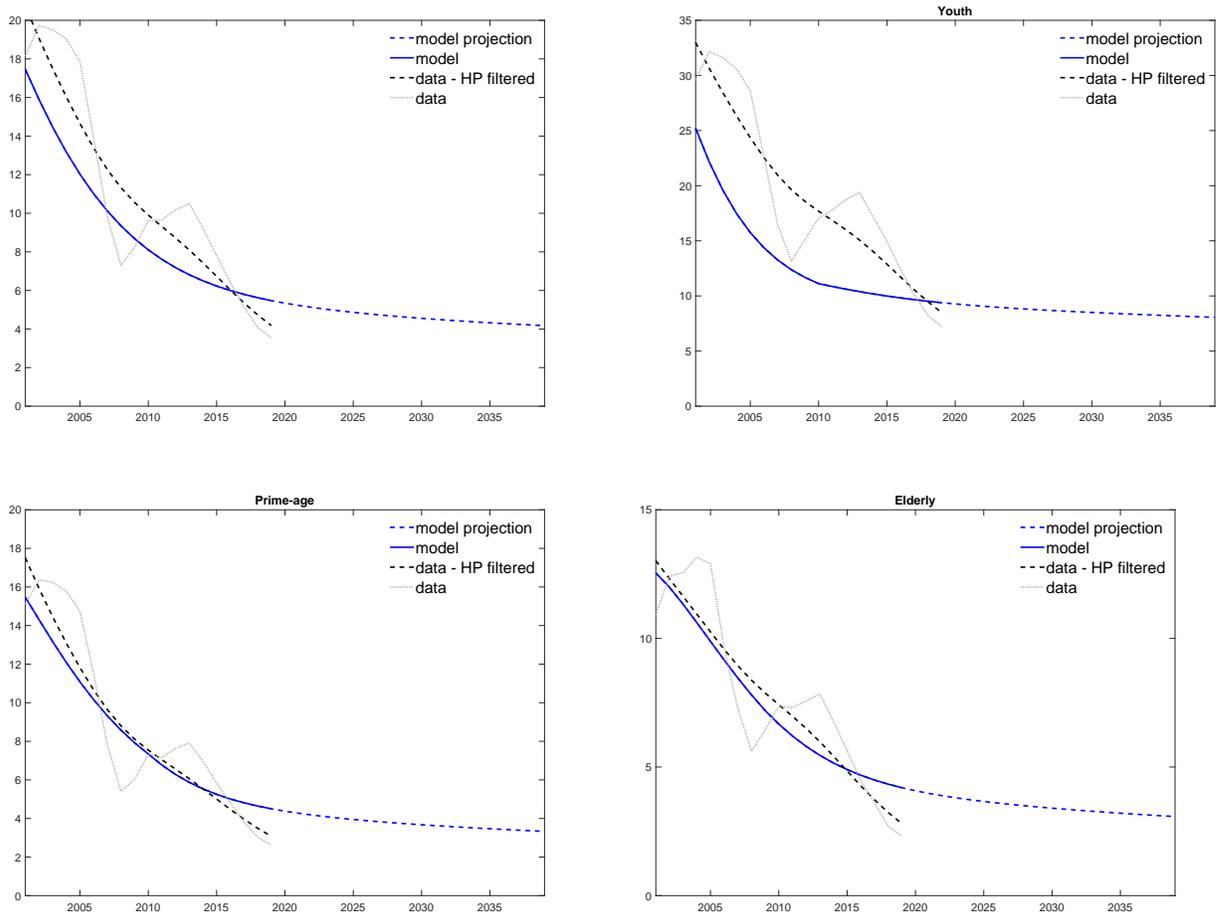


Figure 12: Projection of job find rate rate for young, prime and elderly workers: model vs data and HP trend for Poland

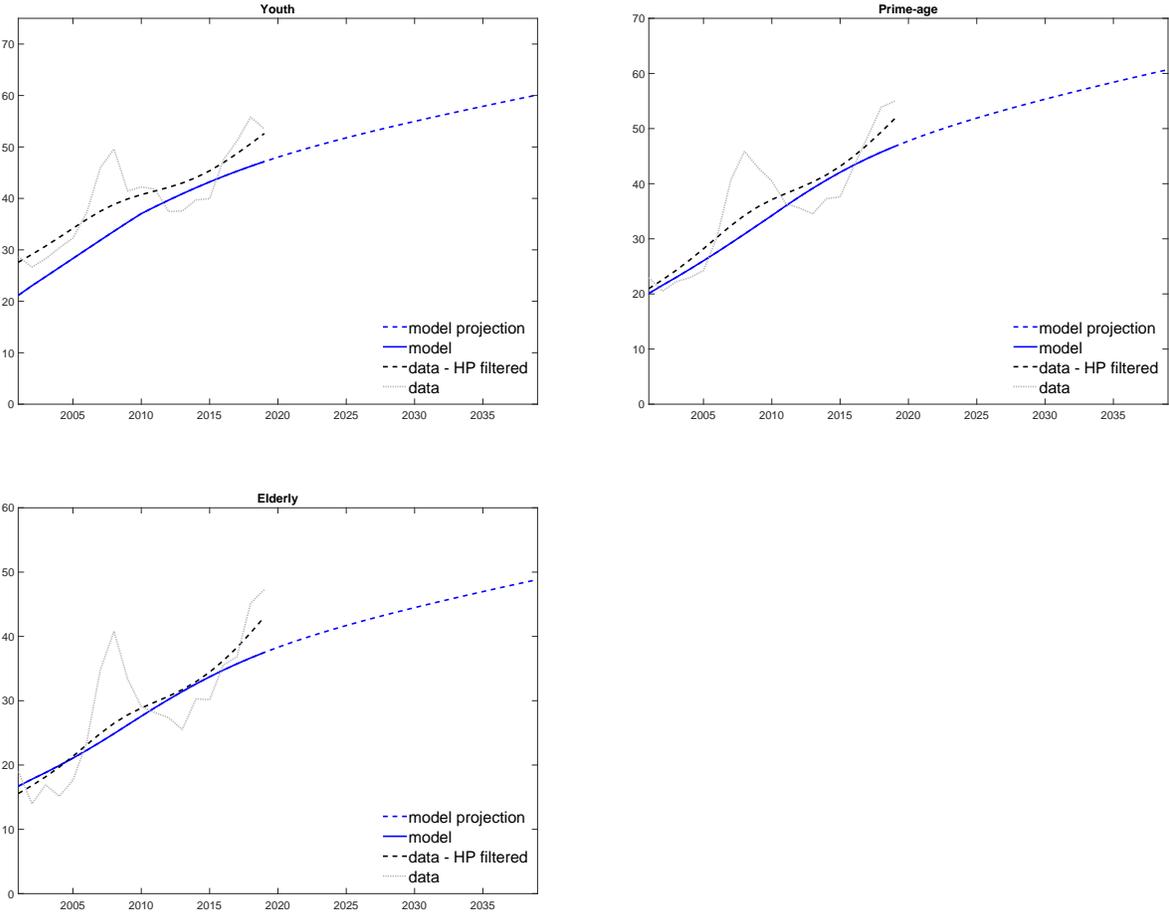


Figure 13: Projection of decomposition of unemployment change for Poland

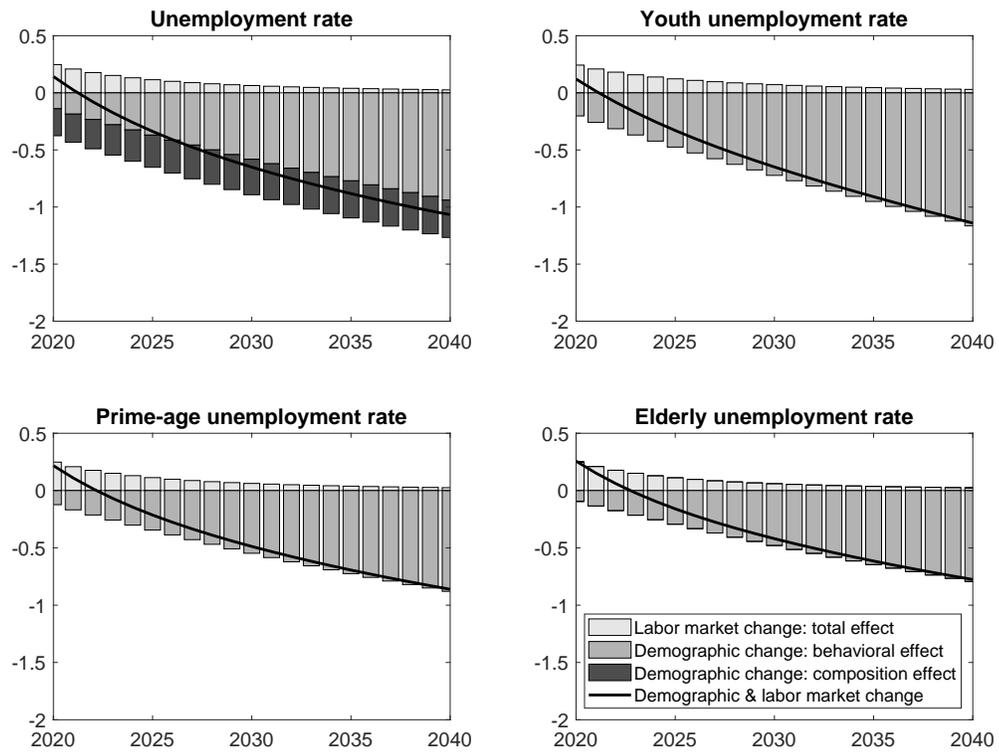


Figure 14: IRFs of monetary policy shock with 2000, 2015 and 2040 population for Poland

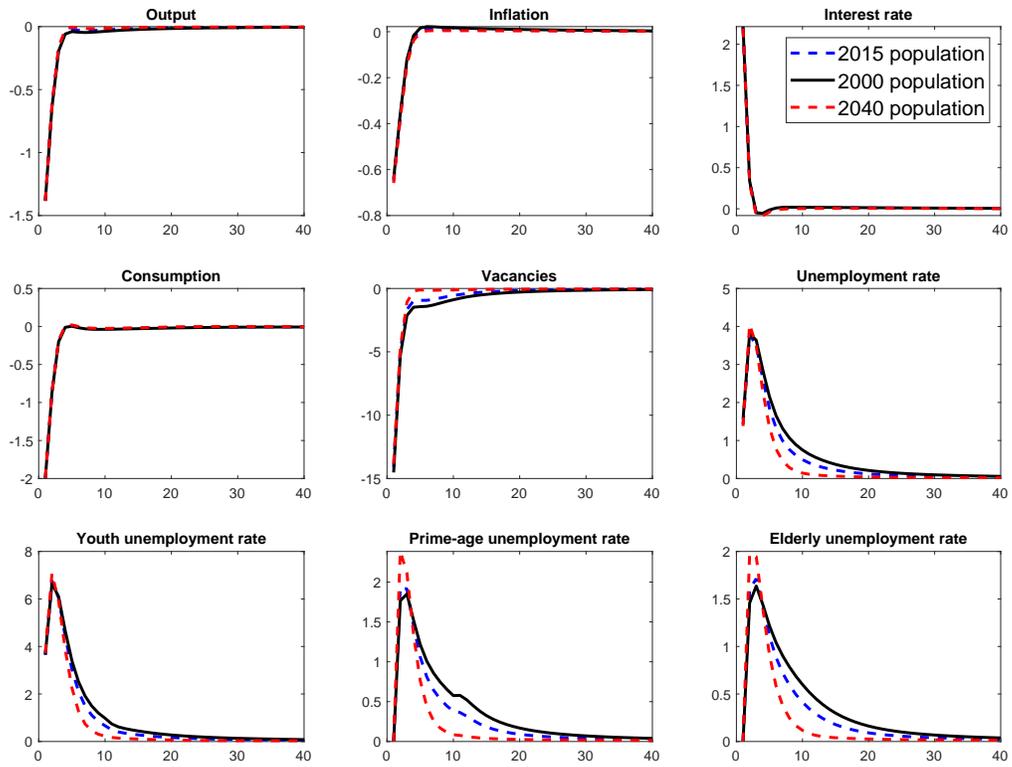


Figure 15: Monetary policy frontier with 2005, 2015 and 2040 population for Poland

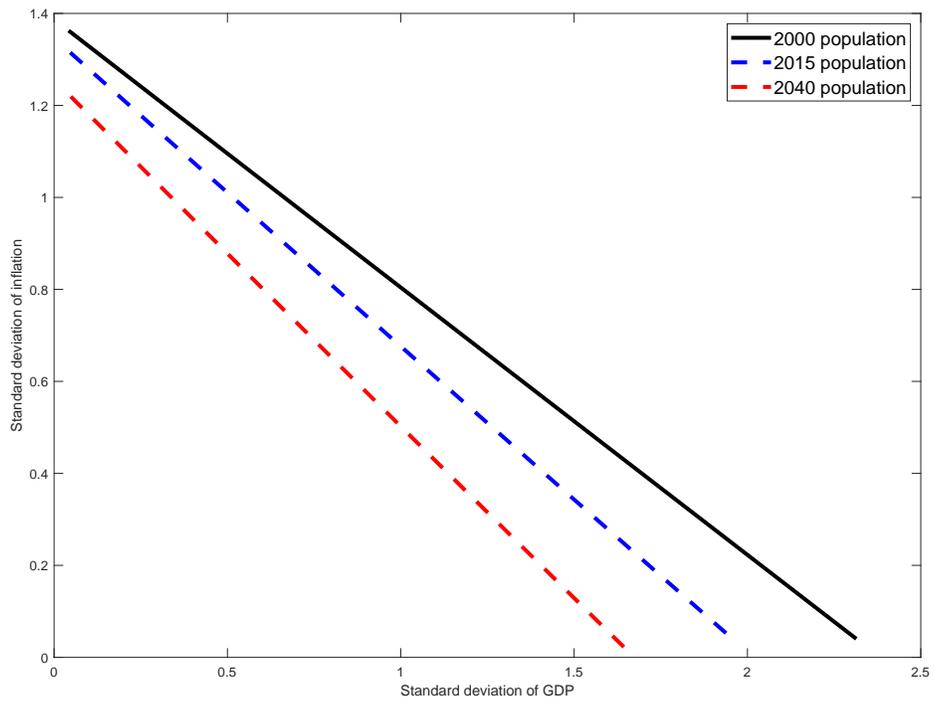


Figure 16: Trade-off at monetary policy frontier in the Poland

